OPERATION OF THE SIMPLEST GAS EJECTOR FROM THE POINT OF VIEW OF THE THERMODYNAMICS OF IRREVERSIBLE PROCESSES

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It is shown that the existence of critical modes of operation of a sonic gas ejector with a cylindrical mixing chamber can be determined by one of the theorems of the thermodynamics of irreversible processes, the Prigogine theorem.

Let us consider from the point of view of the thermodynamics of irreversible processes the operation of a sonic gas ejector with a cylindrical mixing chamber (Fig. 1), with the idea of proving that the existence of critical modes of operation of gas ejectors can be determined by one of the theorems of the thermodynamics of irreversible processes, the Prigogine theorem.

We will assume that 1) the specific heat capacities c_p and c_V of the ejecting and ejected gases and of the gas mixture at the exit from the mixing chamber of the ejector do not depend on the temperature and are identical, 2) the flows of these gases are one-dimensional, 3) the stagnation temperatures of the ejecting and ejected gases are equal, and 4) wall friction and heat transfer through the walls of the mixing chamber are absent.

With these assumptions the geometrical and gas-dynamic parameters of the ejector under consideration are related, as shown in [1-3], by the following equations:

$$T_{03} = T_{01} = T_{02}, \tag{1}$$

$$\frac{p_{03}}{p_{01}} = \frac{\alpha(1+n)}{1+\alpha} \cdot \frac{1}{q(\lambda_3)}, \tag{2}$$

$$\frac{p_{03}}{p_{02}} = \frac{1+n}{n(1+\alpha)} \cdot \frac{q(\lambda_2)}{q(\lambda_3)}, \tag{3}$$

$$z(\lambda_3) = \frac{1+nz(\lambda_2)}{1+n}. \tag{4}$$

In the equations presented, which are the consequence of the laws of conservation of energy, mass, and momentum, T_0 and p_0 are the stagnation temperature and pressure; $\alpha = F_1/F_2$ is the ratio of cross-sectional areas of the ejecting and ejected gases at the entrance to the mixing chamber; $n = G_2/G_1$ is the ratio of mass flow rates of these gases or the ejection coefficient; λ is the velocity coefficient;

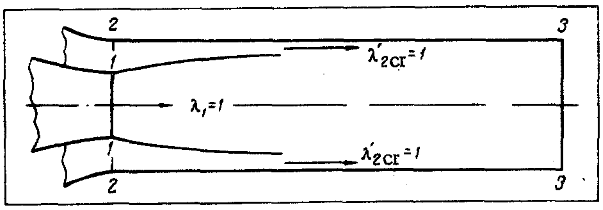

Fig. 1. Schematic diagram of ejector.

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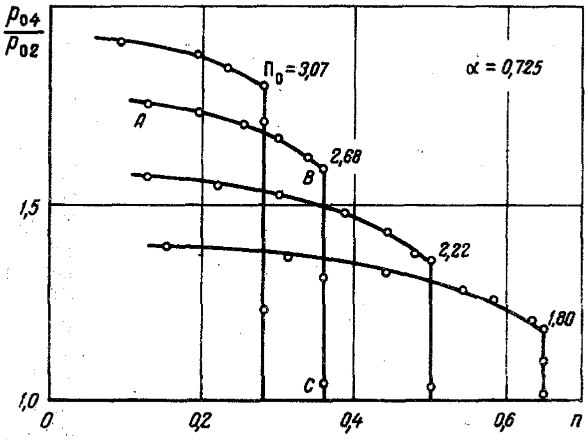

Fig. 2. Typical throttle characteristics of ejector.

$$q(\lambda) = \left(\frac{\varkappa+1}{2}\right)^{\frac{1}{\varkappa-1}} \lambda \left(1-\frac{\varkappa-1}{\varkappa+1}\lambda^2\right)^{\frac{1}{\varkappa-1}},$$

where $\varkappa = c_p/c_v$;

$$z(\lambda) = \frac{1}{2}\left(\lambda + \frac{1}{\lambda}\right) \tag{5}$$

and finally, the indices 1, 2, and 3 denote the parameters of the ejecting and ejected gases at the entrance to the mixing chamber and of the gas mixture at the exit from the mixing chamber of the ejector, respectively (see Fig. 1).

It follows from Eqs. (2) and (3) that

$$q(\lambda_2) = \alpha \Pi_0 n, \tag{6}$$

where $\Pi_0 = p_{01}/p_{02}$ is the ratio of stagnation pressures of the ejecting and ejected gases.

Equation (4), the designation (5), and Eq. (6) show that for fixed values of α, Π_0, and n the conservation laws allow two solutions for the velocity coefficient λ_3:

$$\lambda_3^* = z(\lambda_3) - \sqrt{z^2(\lambda_3) - 1},$$
$$\lambda_3^{**} = z(\lambda_3) + \sqrt{z^2(\lambda_3) - 1}.$$

The first of these solutions corresponds to subsonic and the second to supersonic flow at the exit from the ejector mixing chamber, since

$$\lambda_3^* \lambda_3^{**} = 1.$$

No analytically or physically justified rules for choosing the coefficient λ_3 from λ_3^* or λ_3^{**} exist at present. This leads to the fact that at fixed values of α, Π_0, and p_4, the pressure in the space where the discharge from the ejector mixing chamber occurs, the conservation laws (1)-(4) are satisfied, generally speaking, by an infinite set of values of n.

The results of a test of the ejector under consideration at fixed values of α and Π_0 can be represented in the form of the dependence on n of p_{03}/p_{02}, which is obtained upon a decrease in the pressure p_4. In accordance with these results (see, for example, Fig. 2, borrowed from [4], where p_{04} is the total pressure at the exit from the ejector diffuser) the ejection coefficient n first increases with a decrease in the pressure p_4 and then after reaching some critical value n_{cr} it remains constant. At subcritical modes of ejector operation (corresponding to the sections AB of the dependences of p_{04}/p_{20} on n in Fig. 2) the flow velocity at the exit from the mixing chamber is subsonic, i.e., $\lambda_3 = \lambda_3^*$, while in the supercritical modes (corresponding to sections BC in Fig. 2) it is supersonic, i.e., $\lambda_3 = \lambda_3^{**}$. In the subcritical modes $p_3 = p_4$, while in the supercritical modes $p_3 < p_4$, in connection with which one shock wave or a system of shock waves, in which the pressure p_3 exceeds p_4, develops at the exit from the ejector in these modes.

The existence of critical modes of operation of gas ejectors was discovered experimentally by M. D. Millionshchikov and G. M. Ryabinkov. After the discovery these modes were studied by I. I. Mezhirov

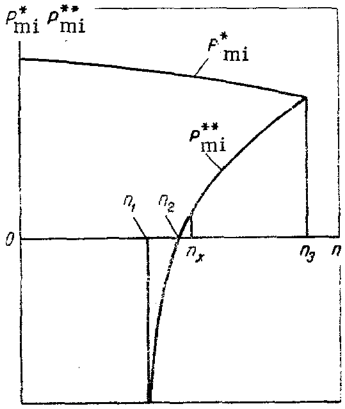

Fig. 3. Form of dependences of P^*_{mi} and P^{**}_{mi} on n.

and G. I. Taganov, A. A. Nikol'skii and V. I. Shustov, Yu. N. Vasil'ev, G. L. Grodzovskii, and other authors in whose reports different methods were proposed for determining n_{cr} which give practically identical results in wide ranges of α, Π_0, etc.

All the methods indicated above are based on the same point of view toward the nature of the critical modes of ejector operation according to which these modes are realized when the ejected gas reaches the speed of sound in some internal section of the mixing chamber (see Fig. 1), usually called the blocking section.

The values of n_{cr} which are obtained by the method suggested in [5] are used below as the values of n_{cr} corresponding to this point of view.

In addition to the simplifying assumptions 1), 2), and 4) formulated above the following assumptions are made in [5]: the flows of the ejecting and ejected gases up to the blocking section do not mix with each other and are one-dimensional; the static pressures of the ejecting and ejected gases at the blocking section differ but are such that between the entrance section and the blocking section the momentum equation is satisfied.

With all these assumptions the values of n_{cr} for a sonic ejector with a cylindrical mixing chamber and $T_{01} = T_{02}$ can be determined from the system of equations

$$
\begin{gathered}
q(\lambda_{2cr}) = \alpha \Pi_0 n_{cr}, \\
q(\lambda'_{1cr}) = \frac{q(\lambda_{2cr})}{n_{cr}\Pi_0 \left[1 - q(\lambda_{2cr})\right] + q(\lambda_{2cr})}, \\
z(\lambda'_{1cr}) = 1 + n_{cr}\left[z(\lambda_{2cr}) - 1\right],
\end{gathered}
\tag{7}
$$

where the index cr designates the critical mode of ejector operation while ' designates the blocking section.

In the case $n_{cr} = 0$ the system (7) becomes indeterminate and therefore it must be transformed again with the help of passage to the limit $n_{cr} = 0$:

$$
\begin{gathered}
q(\lambda'_{1cr}) = \frac{\alpha}{\alpha + 1}, \\
\Pi_0 = \frac{1}{2}\left(\frac{\varkappa + 1}{2}\right)^{\frac{1}{\varkappa - 1}} \cdot \frac{1}{\alpha\left[z(\lambda'_{1cr}) - 1\right]}.
\end{gathered}
\tag{8}
$$

This system determines Π_0 and $n_{cr} = 0$.

It is important to note in addition that the values of n_{cr} determined by system (7) are in satisfactory agreement with the experimental values of n_{cr} in wide ranges of α and Π_0.

After these preliminary but subsequently needed remarks let us introduce into the discussion the concept of the production of entropy in the ejector mixing chamber and then, operating with this concept, we will make use of the second law of thermodynamics and Prigogine's theorem.

In accordance with the definition (see [6], for example), the production of entropy P in any thermodynamic system is the increase per unit time of that part of the entropy which originates in the system itself:

$$
P = \frac{d_i S}{dt}, \tag{9}
$$

where d_iS is the increase in entropy originating in the thermodynamic system in the time dt.

It follows from the definition (9) that the production of entropy in the ejector mixing chamber is

$$
P_{mi} = (G_1 + G_2)\, s_3 - (G_1 s_1 + G_2 s_2), \tag{10}
$$

where s is the entropy of a unit mass of gas and $(G_1 + G_2)s_3$ and $G_1s_1 + G_2s_2$ are the fluxes of entropy at the entrance and exit sections of the mixing chamber, respectively.

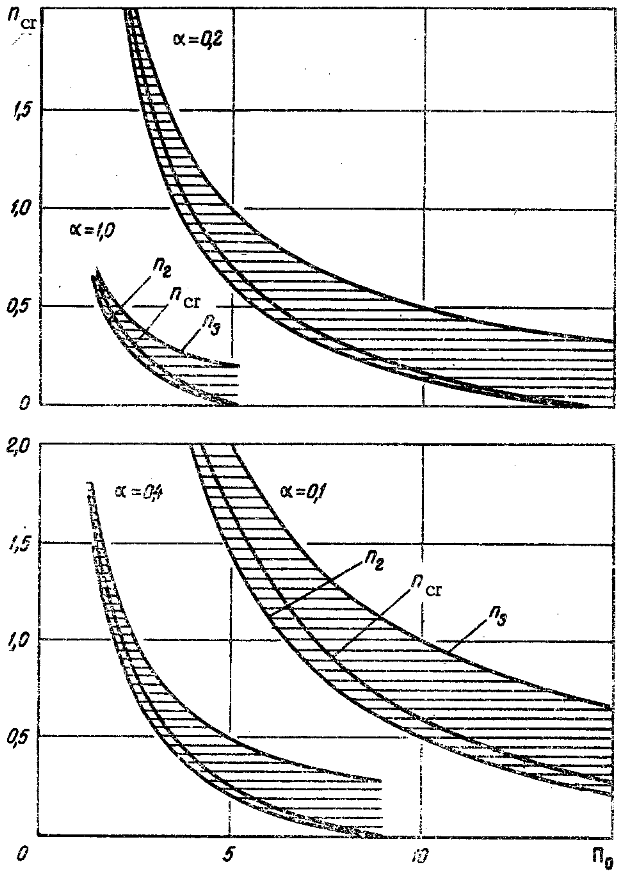

Fig. 4. Dependences of n_{cr}, n_2, and n_3 on Π_0.

As is known, the entropy of a unit mass of an ideal gas with the accuracy of the constant equals

$$s=\frac{1}{\mu}\left(\int C_{p\mu}\frac{dT}{T}-R\ln p\right),$$

where μ is the molecular weight, $C_{p\mu}$ is the molecular heat capacity at constant pressure, and R is the universal gas constant. If $C_{p\mu}$ does not depend on the temperature (and just such a case, as noted above, is considered in the report),

$$s=B\left(\frac{\varkappa}{\varkappa-1}\ln T-\ln p\right)=B\ln\frac{T_0^{\frac{\varkappa}{\varkappa-1}}}{p_0},$$

where B is a specific gas constant.

From this, with allowance for Eqs. (1)-(3), it follows that Eq. (10) can be converted to the form

$$P_{mi}=BG_1\left\{\ln\left[\frac{1+\alpha}{\alpha(1+n)}q(\lambda_3)\right]+n\ln\left[\frac{n(1+\alpha)}{1+n}\frac{q(\lambda_3)}{q(\lambda_2)}\right]\right\}, \tag{11}$$

where the coefficient λ_3 is determined by Eq. (4) and λ_2 by Eq. (6).

Two values of P_{mi} will figure below: P^*_{mi} corresponding to $\lambda_3=\lambda_3^*$ and P^{**}_{mi} corresponding to $\lambda_3=\lambda_3^{**}$.

The calculations show that independently of the values of α and Π_0 the dependences of P^*_{mi} and P^{**}_{mi} on n (see Fig. 3) have the following properties: $P^*_{mi}(n)>0$ if $0\le n\le n_3$, $P^{**}_{mi}(n)=-\infty$ if $n=n_1$, $P^{**}_{mi}(n)\le 0$ if $n_1\le n\le n_2$, $0\le P^{**}_{mi}(n)<P^*_{mi}(n)$ if $n_2\le n<n_3$, $P^*_{mi}(n)=P^{**}_{mi}(n)$ if $n=n_3$.

In these relationships the value n_1 corresponds to the maximum possible value of $z(\lambda_3)=\varkappa/\sqrt{\varkappa^2-1}$ when $\lambda_3>1$, i.e.,

$$n_1=\frac{\varkappa-\sqrt{\varkappa^2-1}}{\sqrt{\varkappa^2-1}\,z(\lambda_2)-\varkappa},$$

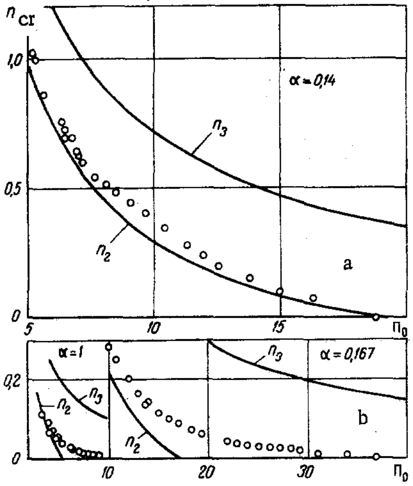

Fig. 5. Comparison of n_2 and n_3 with experimental values of n_{cr}.

where $q(\lambda_2) = \alpha\Pi_0 n_1$. The value n_2 is that value of n at which the entropy production P_{mi}^{**} becomes equal to zero. And finally, the value n_3 is the maximum possible value of the ejection coefficient, corresponding to $\lambda_1 = \lambda_2 = \lambda_3 = 1$:

$$n_3 = \frac{1}{\alpha\Pi_0}. \tag{12}$$

Let us consider the dependences $P_{mi}^{*}(n)$ and $P_{mi}^{**}(n)$ from the point of view of the second law of thermodynamics.

According to the second law the production of entropy both within an entire isolated thermodynamic system and within any of its parts can only be nonnegative. In other words, for the ejector considered here the following conditions must always be satisfied:

$$P_{mi} \geqslant 0, \tag{13}$$

$$\frac{dP_{mi}(l)}{dl} \geqslant 0, \tag{14}$$

where l is the distance between the entrance section and an arbitrary internal section of the mixing chamber and $P_{mi}(l)$ is the entropy production between these sections.

Since $P_{mi}^{*}(n) > 0$ for all values of n while $P_{mi}^{**}(n) \geq 0$ for $n \geq n_2$, in accordance with the condition (13) in the case when $\lambda_3 = \lambda_3^{*}$ all values of n are possible, i.e., $0 \leq n \leq n_3$, while in the case when $\lambda_3 = \lambda_3^{**}$ only values of n equal to or greater than n_2 are possible, i.e., $n_2 \leq n \leq n_3$.

Naturally it is not possible to control in general form the fulfillment of condition (14) when considering only the entrance and exit sections of the ejector mixing chamber. However, it is clear in advance that this condition is not satisfied when $\lambda_3 = \lambda_3^{**}$ and $n = n_2$, and conversely, it is satisfied when $\lambda_3 = \lambda_3^{*}$ or $\lambda_3 = \lambda_3^{**}$ and $n = n_3$. Actually, if condition (14) were satisfied in the first of these cases then this case, on the one hand, could actually be achieved, while on the other hand it would be characterized by zero entropy production during the irreversible process of the mixing of the ejecting and ejected gases. In the second case $\lambda_1 = \lambda_2 = \lambda_3 = 1$, and the entropy production between the entrance section and an arbitrary internal section of the mixing chamber will be the greater, the more completely the process of mixing of the ejecting and ejected gases takes place. Obviously, this process can only develop monotonically along the mixing chamber.

The circumstance that condition (13) is always satisfied while (14) is not satisfied when $\lambda_3 = \lambda_3^{**}$ and $n = n_2$ and is satisfied when $\lambda_3 = \lambda_3^{**}$ and $n = n_3$ allows one to presume the existence of a value n_x starting with which the conditions (13) and (14) are satisfied simultaneously. This value n_x (see Fig. 3) naturally must lie between n_2 and n_3:

$$n_2 < n_x \leqslant n_3.$$

Finally, let us use Prigogine's theorem, according to which the value of P must be the least of all possible values for the given conditions:

$$P = P_{min}.$$

We must note first, however, that the Prigogine theorem is used here in the formulation due to C. Kittel [7]: the stationary, i.e., independent of time, state of the system in which the irreversible process occurs is the state having the least value of entropy production compatible with the external conditions.

Let us examine the process of a decrease in the pressure p_4 and an increase in the ejection coefficient n with fixed values of α and Π_0.

When that pressure p_4 is reached at which the coefficient n becomes equal to n_x, according to Prigogine's theorem a transition should take place from the mode for which $\lambda_3 = \lambda_3^{*}$ to the mode for which $\lambda_3 = \lambda_3^{**}$, since $P_{mi}^{*}(n_x)$ is greater than $P_{mi}^{**}(n_x)$. With a further decrease in p_4 the coefficient n must remain constant according to Prigogine's theorem, since the value of $P_{mi}^{**}(n_x)$ is the smallest possible.

The process of an increase in p_4 with fixed values of α and Π_0 can be examined in an analogous way, as well as the case of the instantaneous establishment of some values of p_4 and Π_0 for a given value of α.

Thus, from the point of view of classical thermodynamics and the thermodynamics of irreversible processes the question of choosing the coefficient λ_3 from the values $\lambda_3 = \lambda_3^*$ and $\lambda_3 = \lambda_3^{**}$ which are allowed by the conservation laws and the question of the nature of the critical mode of operation of the ejector under consideration are interrelated: for fixed values of α and Π_0 the value $\lambda_3 = \lambda_3^{**}$ and the critical mode are realized for the smallest possible value of the entropy production $P_{mi}^{**}(n_x)$.

It follows from the discussion that $n_{cr} = n_x$ and $n_2 < n_{cr} < n_3$.

Here the value n_2 is determined from the condition $P_{mi}^{**}(n_2) = 0$ which, as follows from (11), is equivalent to the system of equations

$$\ln\left[\frac{1+\alpha}{\alpha(1+n_2)}\,q(\lambda_3^{**})\right] + n_2\ln\left[\frac{n_2(1+\alpha)}{1+n_2}\,\frac{q(\lambda_3^{**})}{q(\lambda_2)}\right] = 0.$$

$$z(\lambda_3^{**}) = \frac{1+n_2 z(\lambda_2)}{1+n_2}, \tag{15}$$

$$q(\lambda_2) = \alpha\Pi_0 n_2.$$

For small values of n_2 it is inconvenient to use this system, like the system (7), since at $n_2 = 0$ the product $n_2 z(\lambda_2)$ is converted to indeterminacy of the type $0\cdot\infty$. In connection with this, in addition to system (15) it is advisable to have a system obtained from (15) after transition to the limit $n_2 = 0$:

$$q(\lambda_3^{**}) = \frac{\alpha}{1+\alpha},$$

$$\Pi_0 = \frac{1}{2}\left(\frac{\varkappa+1}{2}\right)^{\frac{1}{\varkappa-1}} \cdot \frac{1}{\alpha\left[z(\lambda_3^{**})-1\right]}. \tag{16}$$

This system determines Π_0 for a given α in the case $n_2 = 0$.

The value n_3 is determined from Eq. (12):

$$n_3 = \frac{1}{\alpha\Pi_0}.$$

Now the question naturally arises of whether the values of n_{cr} which are determined by the system of equations (7) and are in satisfactory agreement, as noted above, with the corresponding experimental values, lie between the values n_2 and n_3 which are determined by the system (15) and Eq. (12).

For an answer to this question the dependences $n_{cr}(\Pi_0)$, $n_2(\Pi_0)$, and $n_3(\Pi_0)$ were calculated for different values of α.

As follows from Fig. 4, where these dependences are presented for several α, the values n_{cr} lie between n_2 and n_3, with n_{cr} only slightly exceeding n_2 especially at large values of α and at small and large values of Π_0. (The maximum values of Π_0 corresponding to $n_{cr} = 0$ and $n_2 = 0$ are the same, since the systems of equations (8) and (16) convert to one another after the substitutions $\lambda'_{1cr} = \lambda_3^{**}$ and $\lambda_3^{**} = \lambda'_{1cr}$.)

It must be noted that the dependences $n_{cr}(\Pi_0)$ and $n_2(\Pi_0)$ for $\alpha = 0.4$ and the dependence $n_2(\Pi_0)$ for $\alpha = 1.0$ are presented in Fig. 4 for those values of n and Π_0 for which the condition of criticality of the discharge of the ejecting gas into the mixing chamber is satisfied:

$$\pi(1)\,\Pi_0 \geqslant \pi(\lambda_2),$$
$$q(\lambda_2) = \alpha\Pi_0 n, \tag{17}$$

where

$$\pi(\lambda) = \left(1 - \frac{\varkappa-1}{\varkappa+1}\,\lambda^2\right)^{\frac{\varkappa}{\varkappa-1}}.$$

The dependence $n_{cr}(\Pi_0)$ for $\alpha = 1.0$ is presented up to the critical value of Π_0 in the case of the corresponding dependence $n_2(\Pi_0)$ since in the case of the same dependence $n_{cr}(\Pi_0)$ at this value of α the condition (17) is always satisfied.

A comparison of the values n_{cr} with the values n_2 and n_3 shows that the existence of critical modes of operation of sonic gas ejectors with cylindrical chambers actually can be determined by Prigogine's theorem.

A comparison of the values n_2 and n_3 with the experimental values of n_{cr} obtained by I. I. Mezhirov and G. I. Taganov (Fig. 5a) and by G. L. Grodzovskii and A. F. Ravdin (Fig. 5b) leads to the same conclusion.

The first of these experimental values were obtained in the case of a peripheral location of the ejecting jet and the second in the case of a central location, and while the first values agree well with the corresponding calculated values of Yu. N. Vasil'ev down to $n_{cr} = 0$ the second values differ considerably from them in the region of large Π_0.

NOTATION

c_p, c_v	are the specific heat capacities at constant pressure and constant volume;
T_0, p_0	are the stagnation temperature and pressure;
α	is the ratio of cross-sectional areas of ejecting and ejected gas flows at entrance to mixing chamber;
n	is the ejection coefficient;
λ	is the velocity coefficient;
Π_0	is the ratio of stagnation pressures of ejecting and ejected gases;
p	is the static pressure;
P	is the entropy production;
S	is the total entropy of system;
t	is the time;
P_{mi}	is the entropy production in ejector mixing chamber;
s	is the entropy of a unit mass of gas;
G	is the mass flow rate;
B	is the specific gas constant;

Subscripts and Superscripts

1, 2, 3	denote parameters of the ejecting and ejected gases at the entrance to the mixing chamber and of the gas mixture at the exit from the mixing chamber of the ejector, respectively;
4	denotes parameters in the space where discharge from the mixing chamber occurs;
cr	denotes the critical mode of ejector operation;
'	denotes the blocking section;
*, **	denote the cases when $\lambda_3 < 1$ and $\lambda_3 > 1$, respectively.

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